(UNIVERSITY OF MUMBAI)

Syllabus for: S.Y.B.Sc./S.Y.B.A.

Program: B.Sc./B/A.
Course: Mathematics
Choice based Credit System (CBCS)
with effect from the
academic year 2018-19

SEMESTER III

CALCULUS III				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 301, UAMT 301	I	Functions of several variables	2	3
	II	Differentiation		
	III	Applications		
ALGEBRA III				
USMT 302 ,UAMT 302	I	Linear Transformations and Matrices	2	3
	II	Determinants		
	III	Inner Product Spaces		
DISCRETE MATHEMATICS				
USMT 303	I	Permutations and Recurrence Relation	2	3
	II	Preliminary Counting		
	III	Advanced Counting		
PRACTICALS				
USMTP03		Practicals based on USMT301, USMT 302 and USMT 303	3	5
UAMTP03		Practicals based on UAMT301, UAMT 302	2	4

SEMESTER IV

CALCULUS IV				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 401, UAMT 401	I	Riemann Integration	2	3
	II	Indefinite Integrals and Improper Integrals		
	III	Beta and Gamma Functions And Applications		
ALGEBRA IV				
USMT 402 ,UAMT 402	I	Groups and Subgroups	2	3
	II	Cyclic Groups and Cyclic subgroups		
	III	Lagrange's Theorem and Group Homomorphism		
ORDINARY DIFFERENTIAL EQUATIONS				
USMT 403	I	First order First degree Differential equations	2	3
	II	Second order Linear Differential equations		
	III	Linear System of Ordinary Differential Equations		
PRACTICALS				
USMTP04		Practicals based on USMT401, USMT 402 and USMT 403	3	5
UAMTP04		Practicals based on UAMT401, UAMT 402	2	4

Teaching Pattern for Semester III

1. Three lectures per week per course. Each lecture is of 48 minutes duration.
2. One Practical (2L) per week per batch for courses USMT301, USMT 302 combined and one Practical (3L) per week for course USMT303 (the batches tobe formed as prescribed by the University. Each practical session is of 48 minutes duration.)

Teaching Pattern for Semester IV

1. Three lectures per week per course. Each lecture is of 48 minutes duration.
2. One Practical (2L) per week per batch for courses USMT301, USMT 302 combined and one Practical (3L) per week for course USMT303 (the batches tobe formed as prescribed by the University. Each practical session is of 48 minutes duration.)

S.Y.B.Sc. / S.Y.B.A. Mathematics

SEMESTER III

USMT 301, UAMT 301: CALCULUS III

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

Unit I: Functions of several variables (15 Lectures)

1. The Euclidean inner product on $\mathbb{R}^n$ and Euclidean norm function on $\mathbb{R}^n$, distance between two points, open ball in $\mathbb{R}^n$, definition of an open subset of $\mathbb{R}^n$, neighbourhood of a point in $\mathbb{R}^n$, sequences in $\mathbb{R}^n$, convergence of sequences- these concepts should be specifically discussed for $n = 3$ and $n = 3$.
2. Functions from $\mathbb{R}^n \longrightarrow \mathbb{R}$ (scalar fields) and from $\mathbb{R}^n \longrightarrow \mathbb{R}^m$ (vector fields), limits, continuity of functions, basic results on limits and continuity of sum, difference, scalar multiples of vector fields, continuity and components of a vector fields.
3. Directional derivatives and partial derivatives of scalar fields.
4. Mean value theorem for derivatives of scalar fields.

Reference for Unit I:
Sections 8.1, 8.2, 8.3, 8.4, 8.5, 8.6, 8.7, 8.8, 8.9, 8.10 of Calculus, Vol. 2 (Second Edition) by Apostol.

Unit II: Differentiation (15 Lectures)

1. Differentiability of a scalar field at a point of $\mathbb{R}^n$ (in terms of linear transformation) and on an open subset of $\mathbb{R}^n$, the total derivative, uniqueness of total derivative of a differentiable function at a point, simple examples of finding total derivative of functions such as $f(x, y) = x^2 + y^2, f(x, y, z) = x + y + z,$, differentiability at a point of a function f implies continuity and existence of direction derivatives of f at the point, the existence of continuous partial derivatives in a neighbourhood of a point implies differentiability at the point.

2. Gradient of a scalar field, geometric properties of gradient, level sets and tangent planes.

3. Chain rule for scalar fields.

4. Higher order partial derivatives, mixed partial derivatives, sufficient condition for equality of mixed partial derivative.

Reference for Unit II:
Sections 8.11, 8.12, 8.13, 8.14, 8.15, 8.16, 8.17, 8.23 of Calculus, Vol.2 (Second Edition) by T. Apostol, John Wiley.

Unit III: Applications (15 lectures)

1. Second order Taylor's formula for scalar fields.

2. Differentiability of vector fields, definition of differentiability of a vector field at a point, Jacobian matrix, differentiability of a vector field at a point implies continuity. The chain rule for derivative of vector fields (statements only)

3. Mean value inequality.

4. Hessian matrix, Maxima, minima and saddle points.

5. Second derivative test for extrema of functions of two variables.

6. Method of Lagrange Multipliers.

Reference for Unit III:
Sections 8.18, 8.19, 8.20, 8.21, 8.22, 9.9, 9.10, 9.11, 9.12, 9.13, 9.14 9.13, 9.14 from Apostol, Calculus Vol. 2, (Second Edition) by T. Apostol.

Recommended Text Books:

1. T. Apostol: Calculus, Vol. 2, John Wiley.

2. J. Stewart, Calculus, Brooke/ Cole Publishing Co.

Additional Reference Books

(1) G.B. Thoman and R. L. Finney, Calculus and Analytic Geometry, Ninth Edition, Addison-Wesley, 1998.

(2) Sudhir R. Ghorpade and Balmohan V. Limaye, A Course in Multivariable Calculus and Analysis, Springer International Edition.

(3) Howard Anton, Calculus- A new Horizon, Sixth Edition, John Wiley and Sons Inc, 1999.

USMT 302/UAMT 302: ALGEBRA III

Note: Revision of relevant concepts is necessary.

Unit 1: Linear Transformations and Matrices (15 lectures)

SEMESTER IV

USMT 401/UAMT 401: CALCULUS IV

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

Unit I: Riemann Integration (15 Lectures)

Approximation of area, Upper/Lower Riemann sums and properties, Upper/Lower integrals, Definition of Riemann integral on a closed and bounded interval, Criterion of Riemann integrability, if $a < c < b$ then $f \in R[a,b]$, if and only if $f \in R[a,c]$ and $f \in R[c,b]$ and $\int_a^b f = \int_a^c f + \int_c^b f$.

Properties:

(i) $f, g \in R[a,b] \Longrightarrow f+g, \lambda f \in R[a,b]$.

(ii) $\int_a^b (f+g) = \int_a^b f + \int_a^b g$.

(iii) $\int_a^b \lambda f = \lambda \int_a^b f$.

(iv) $f \in R[a,b] \Longrightarrow |f| \in R[a,b]$ and $|\int_a^b f| \leq \int_a^b |f|$,

(v) $f \geq 0, f \in C[a,b] \Longrightarrow f \in R[a,b]$.

(vi) If f is bounded with finite number of discontinuities then $f \in R[a,b]$, generalize this if f is monotone then $f \in R[a,b]$.

Unit II: Indefinite and improper integrals (15 lectures)

Continuity of $F(x) = \int_a^x f(t)\ dt$ where $f \in R[a,b]$, Fundamental theorem of calculus, Mean value theorem, Integration by parts, Leibnitz rule, Improper integrals-type 1 and type 2, Absolute convergence of improper integrals, Comparison tests, Abel's and Dirichlet's tests.

Unit III: Applications (15 lectures)

(1) β and Γ functions and their properties, relationship between β and Γ functions (without proff).

(2) Applications of definite Integras: Area between curves, finding volumes by sicing, volumes of solids of revolution-Disks and Washers, Cylindrical Shells, Lengths of plane curves, Areas of surfaces of revolution.

References:

(1) Calculus Thomas Finney, ninth edition section 5.1, 5.2, 5.3, 5.4, 5.5, 5.6.

(2) R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.

(3) Ajit Kumar, S.Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
(4) T. Apostol, Calculus Vol.2, John Wiley.
(5) K. Stewart, Calculus, Booke/Cole Publishing Co, 1994.
(6) J. E. Marsden, A.J. Tromba and A. Weinstein, Basic multivariable calculus.
(7) Bartle and Sherbet, Real analysis.

USMT 402/ UAMT 402: ALGEBRA IV

Unit I: Groups and Subgroups (15 Lectures)

(a) Definition of a group, abelian group, order of a group, finite and infinite groups. Examples of groups including:

- i) $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$ under addition.
- ii) $\mathbb{Q}^*(= \mathbb{Q} \setminus \{0\}), \mathbb{R}^*(= \mathbb{R} \setminus \{0\}), \mathbb{C}^*(= \mathbb{C} \setminus \{0\}).\mathbb{Q}^+$(= positive rational numbers) under multiplication.
- iii) $\mathbb{Z}_n$, the set of residue classes modulo n under addition.
- iv) $U(n)$, the group of prime residue classes modulo n under multiplication.
- v) The symmetric group S_n.
- vi) The group of symmetries of a plane figure. The Dihedral group D_n as the group of symmetries of a regular polygon of n sides (for $n = 3, 4$).
- vii) Klein 4-group.
- viii) Matrix groups $M_{n\times n}(\mathbb{R})$ under addition of matrices, $GL_n(\mathbb{R})$, the set of invertible real matrices, under multiplication of matrices.
- ix) Examples such as S^1 as subgroup of C, μ_n the subgroup of $n-$th roots of unity.

(b) Properties such as

1) In a group $(G, .)$ the following indices rules are true for all integers n, m.
   - i) $a^n a^m = a^{n+m}$ for all a in G.
   - ii) $(a^n)^m = a^{nm}$ for all a in G.
   - iii) $(ab)^n = a^n b^n$ for all ab in G whenever $ab = ba$.
2) In a group $(G, .)$ the following are true:
   - i) The identity element e of G is unique.
   - ii) The inverse of every element in G is unique.
   - iii) $(a^{-1})^{-1} = a$ for all a in G.
   - iv) $(a.b)^{-1} = b^{-1}a^{-1}$ for all a, b in G.
   - v) If $a^2 = e$ for every a in G then $(G, .)$ is an abelian group.
   - vi) $(aba^{-1})^n = ab^n a^{-1}$ for every a, b in G and for every integer n.
   - vii) If $(a.b)^2 = a^2.b^2$ for every a, b in G then $(G, .)$ is an abelian group.
   - viii) $(\mathbb{Z}_n^*, .)$ is a group if and only if n is a prime.
3) Properties of order of an element such as: (n and m are integers.)
   - i) If $o(a) = n$ then $a^m = e$ if and only if n/m.
   - ii) If $o(a) = nm$ then $o(a^n) = m$.
   - iii) If $o(a) = n$ then $o(a^m) = \dfrac{n}{(n, m)}$,. where (n, m) is the GCD of n and m.

6. Group homomorphisms, isomorphisms.
7. Miscellaneous Theoretical questions based on full paper.

Suggested Practicals for USMT403:

1. Solving exact and non exact equations.
2. Linear and reducible to linear equations, applications to orthogonal trajectories, population growth, and finding the current at a given time.
3. Finding general solution of homogeneous and non-homogeneous equations, use of known solutions to find the general solution of homogeneous equations.
4. Solving equations using method of undetermined coefficients and method of variation of parameters.
5. Solving second order linear ODEs
6. Solving a system of first order linear ODES.
7. Miscellaneous Theoretical questions from all units.

Scheme of Examination

I. **Semester End Theory Examinations:** There will be a Semester-end external Theory examination of 100 marks for each of the courses USMT301/UAMT301, USMT302/UAMT302, USMT303 of Semester III and USMT401/UAMT401, USMT402/UAMT402, USMT403 of semester IV to be conducted by the University.

   1. Duration: The examinations shall be of 3 Hours duration.
   2. Theory Question Paper Pattern:
      a) There shall be FIVE questions. The first question Q1 shall be of objective type for 20 marks based on the entire syllabus. The next three questions Q2, Q2, Q3 shall be of 20 marks, each based on the units I, II, III respectively. The fifth question Q5 shall be of 20 marks based on the entire syllabus.
      b) All the questions shall be compulsory. The questions Q2, Q3, Q4, Q5 shall have internal choices within the questions. Including the choices, the marks for each question shall be 30-32.
      c) The questions Q2, Q3, Q4, Q5 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.
      d) The question Q1 may be subdivided into 10 sub-questions of 2 marks each.

II. **Semester End Examinations Practicals:**
At the end of the Semesters III and IV, Practical examinations of three hours duration and 150 marks shall be conducted for the courses USMTP03, USMTP04.

At the end of the Semesters III and IV, Practical examinations of three hours duration and 150 marks shall be conducted for the courses UAMTP03, UAMTP04.

In semester III, the Practical examinations for USMT301/UAMT301 and USMT302/UAMT302 are held together by the college. The Practical examination for USMT303 is held **separately** by the college.

In semester IV, the Practical examinations for USMT401/UAMT401 and USMT402/UAMT402 are held together by the college. The Practical examination for USMT403 is held **separately** by the college.

Paper pattern: The question paper shall have three parts A, B, C.
Each part shall have two Sections.

Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions. ($8 \times 3 = 24$ Marks)

Section II Problems: Attempt any Two out of Three. ($8 \times 2 = 16$ Marks)

Practical Course	Part A	Part B	**Part C**	Marks out of	duration
USMTP03	Questions from USMT301	Questions from USMT302	Questions from USMT303	120	3 hours
UAMTP03	Questions from UAMT301	Questions from UAMT302	—	80	2 hours
USMTP04	Questions from USMT401	Questions from USMT402	Questions from USMT403	120	3 hours
UAMTP03	Questions from UAMT401	Questions from UAMT402	—	80	2 hours

Marks for Journals and Viva:
For each course USMT301/UAMT301, USMT302/UAMT302, USMT303, USMT401/UAMT401, USMT402/UAMT402 and USMT403:

1. Journals: 5 marks.
2. Viva: 5 marks.

Each Practical of every course of Semester III and IV shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal. A student must have a certified journal before appearing for the practical examination.