(UNIVERSITY OF MUMBAI)

Syllabus for: S.Y.B.Sc./S.Y.B.A.

Program: B.Sc./B/A.
Course: Mathematics
Choice based Credit System (CBCS)
with effect from the
academic year 2018-19

SEMESTER III

CALCULUS III				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 301, UAMT 301	I	Functions of several variables	2	3
	II	Differentiation		
	III	Applications		
ALGEBRA III				
USMT 302 ,UAMT 302	I	Linear Transformations and Matrices	2	3
	II	Determinants		
	III	Inner Product Spaces		
DISCRETE MATHEMATICS				
USMT 303	I	Permutations and Recurrence Relation	2	3
	II	Preliminary Counting		
	III	Advanced Counting		
PRACTICALS				
USMTP03		Practicals based on USMT301, USMT 302 and USMT 303	3	5
UAMTP03		Practicals based on UAMT301, UAMT 302	2	4

SEMESTER IV

CALCULUS IV				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 401, UAMT 401	I	Riemann Integration	2	3
	II	Indefinite Integrals and Improper Integrals		
	III	Beta and Gamma Functions And Applications		
ALGEBRA IV				
USMT 402 ,UAMT 402	I	Groups and Subgroups	2	3
	II	Cyclic Groups and Cyclic subgroups		
	III	Lagrange's Theorem and Group Homomorphism		
ORDINARY DIFFERENTIAL EQUATIONS				
USMT 403	I	First order First degree Differential equations	2	3
	II	Second order Linear Differential equations		
	III	Linear System of Ordinary Differential Equations		
PRACTICALS				
USMTP04		Practicals based on USMT401, USMT 402 and USMT 403	3	5
UAMTP04		Practicals based on UAMT401, UAMT 402	2	4

Teaching Pattern for Semester III

1. Three lectures per week per course. Each lecture is of 48 minutes duration.
2. One Practical (2L) per week per batch for courses USMT301, USMT 302 combined and one Practical (3L) per week for course USMT303 (the batches tobe formed as prescribed by the University. Each practical session is of 48 minutes duration.)

Teaching Pattern for Semester IV

1. Three lectures per week per course. Each lecture is of 48 minutes duration.
2. One Practical (2L) per week per batch for courses USMT301, USMT 302 combined and one Practical (3L) per week for course USMT303 (the batches tobe formed as prescribed by the University. Each practical session is of 48 minutes duration.)

S.Y.B.Sc. / S.Y.B.A. Mathematics

SEMESTER III

USMT 301, UAMT 301: CALCULUS III

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

Unit I: Functions of several variables (15 Lectures)

1. The Euclidean inner product on $\mathbb{R}^n$ and Euclidean norm function on $\mathbb{R}^n$, distance between two points, open ball in $\mathbb{R}^n$, definition of an open subset of $\mathbb{R}^n$, neighbourhood of a point in $\mathbb{R}^n$, sequences in $\mathbb{R}^n$, convergence of sequences- these concepts should be specifically discussed for $n = 3$ and $n = 3$.
2. Functions from $\mathbb{R}^n \longrightarrow \mathbb{R}$ (scalar fields) and from $\mathbb{R}^n \longrightarrow \mathbb{R}^m$ (vector fields), limits, continuity of functions, basic results on limits and continuity of sum, difference, scalar multiples of vector fields, continuity and components of a vector fields.
3. Directional derivatives and partial derivatives of scalar fields.
4. Mean value theorem for derivatives of scalar fields.

Reference for Unit I:
Sections 8.1, 8.2, 8.3, 8.4, 8.5, 8.6, 8.7, 8.8, 8.9, 8.10 of Calculus, Vol. 2 (Second Edition) by Apostol.

Unit II: Differentiation (15 Lectures)

1. Differentiability of a scalar field at a point of $\mathbb{R}^n$ (in terms of linear transformation) and on an open subset of $\mathbb{R}^n$, the total derivative, uniqueness of total derivative of a differentiable function at a point, simple examples of finding total derivative of functions such as $f(x, y) = x^2 + y^2, f(x, y, z) = x + y + z,$, differentiability at a point of a function f implies continuity and existence of direction derivatives of f at the point, the existence of continuous partial derivatives in a neighbourhood of a point implies differentiability at the point.

2. Norm of a vector in an inner product space. Cauchy-Schwartz inequality, Triangle inequality, Orthogonality of vectors, Pythagoras theorem and geometric applications in $\mathbb{R}^2$, Projections on a line, The projection being the closest approximation, Orthogonal complements of a subspace, Orthogonal complements in $\mathbb{R}^2$ and $\mathbb{R}^3$. Orthogonal sets and orthonormal sets in an inner product space, Orthogonal and orthonormal bases. Gram-Schmidt orthogonalization process, Simple examples in $\mathbb{R}^3, \mathbb{R}^4$.

Reference of Unit 3: Chapter VI, Sections 1,2 of Introduction to Linear Algebra, Serge Lang, Springer Verlag and Chapter 5, of Linear Algebra A Geometric Approach, S. Kumaresan, Prentice-Hall of India Private Limited, New Delhi.

Recommended Books:

1. Serge Lang: Introduction to Linear Algebra, Springer Verlag.
2. S. Kumaresan: Linear Algebra A geometric approach, Prentice Hall of India Private Limited.

Additional Reference Books:

1. M. Artin: Algebra, Prentice Hall of India Private Limited.
2. K. Hoffman and R. Kunze: Linear Algebra, Tata McGraw-Hill, New Delhi.
3. Gilbert Strang: Linear Algebra and its applications, International Student Edition.
4. L. Smith: Linear Algebra, Springer Verlag.
5. A. Ramachandra Rao and P. Bhima Sankaran: Linear Algebra, Tata McGraw-Hill, New Delhi.
6. T. Banchoff and J. Wermer: Linear Algebra through Geometry, Springer Verlag Newyork, 1984.
7. Sheldon Axler: Linear Algebra done right, Springer Verlag, Newyork.
8. Klaus Janich: Linear Algebra.
9. Otto Bretcher: Linear Algebra with Applications, Pearson Education.
10. Gareth Williams: Linear Algebra with Applications, Narosa Publication.

USMT 303: Discrete Mathematics

Unit I: Permutations and Recurrence relation (15 lectures)

1. Permutation of objects, S_n, composition of permutations, results such as every permutation is a product of disjoint cycles, every cycle is a product of transpositions, even and odd permutation, rank and signature of a permutation, cardinality of S_n, A_n
2. Recurrence Relations, definition of non-homogeneous, non-homogeneous, linear , non-linear recurrence relation, obtaining recurrence relation in counting problems, solving homogeneous as well as non homogeneous recurrence relations by using iterative methods, solving a homogeneous recurrence relation of second degree using algebraic method proving the necessary result.

Recommended Books:

1. Norman Biggs: Discrete Mathematics, Oxford University Press.
2. Richard Brualdi: Introductory Combinatorics, John Wiley and sons.
3. V. Krishnamurthy: Combinatorics-Theory and Applications, Affiliated East West Press.
4. Discrete Mathematics and its Applications, Tata McGraw Hills.
5. Schaum's outline series: Discrete mathematics,
6. Applied Combinatorics: Allen Tucker, John Wiley and Sons.

Unit II: Preliminary Counting (15 Lectures)

1. Finite and infinite sets, countable and uncountable sets examples such as $\mathbb{N}, \mathbb{Z}, \mathbb{N} \times \mathbb{N}, \mathbb{Q}, (0,1), \mathbb{R}$
2. Addition and multiplication Principle, counting sets of pairs, two ways counting.
3. Stirling numbers of second kind. Simple recursion formulae satisfied by $S(n,k)$ for $k = 1, 2, \cdots, n-1, n$
4. Pigeonhole principle and its strong form, its applications to geometry, monotonic sequences etc.

Unit III: Advanced Counting (15 Lectures)

1. Binomial and Multinomial Theorem, Pascal identity, examples of standard identities such as the following with emphasis on combinatorial proofs.
   - $\sum_{k=0}^{r} \binom{m}{k} \binom{n}{r-k} = \binom{m+n}{r}$
   - $\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$
   - $\sum_{i=0}^{k} \binom{k}{i}^2 = \binom{2k}{k}$
   - $\sum_{i=0}^{n} \binom{n}{i} = 2^n$
2. Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems.
3. Non-negative and positive solutions of equation $x_1 + x_2 + \cdots + x_k = n$
4. Principal of inclusion and exclusion, its applications, derangements, explicit formula for d_n, deriving formula for Euler's function $\phi(n)$.

USMT 403: ORDINARY DIFFERENTIAL EQUATIONS

Unit I: First order First degree Differential equations (15 Lectures)

(1) Definition of a differential equation, order, degree, ordinary differential equation and partial differential equation, linear and non linear ODE.

(2) Existence and Uniqueness Theorem for the solution of a second order initial value problem (statement only), Definition of Lipschitz function, Examples based on verifying the conditions of existence and uniqueness theorem

(3) Review of Solution of homogeneous and non-homogeneous differential equations of first order and first degree. Notion of partial derivatives. Exact Equations: General solution of Exact equations of first order and first degree. Necessary and sufficient condition for $Mdx + Ndy = 0$ to be exact. Non-exact equations: Rules for finding integrating factors (without proof) for non exact equations, such as :

i) $\dfrac{1}{M\ x + N\ y}$ is an I.F. if $M\ x + N\ y \neq 0$ and $Mdx + Ndy = 0$ is homogeneous.

ii) $\dfrac{1}{M\ x - N\ y}$ is an I.F. if $M\ x - N\ y \neq 0$ and $Mdx + Ndy = 0$ is of the form $f_1(x, y)\ y\ dx + f_2(x, y)\ x\ dy = 0$.

iii) $e^{\int f(x)\ dx}$ (resp $e^{\int g(y)\ dy}$) is an I.F. if $N \neq 0$ (resp $M \neq 0$) and $\dfrac{1}{N}\left(\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}\right)$ $\left(\text{resp}\dfrac{1}{M}\left(\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}\right)\right)$ is a function of x (resp y) alone, say $f(x)$ (resp $g(y)$).

iv) Linear and reducible linear equations of first order, finding solutions of first order differential equations of the type for applications to orthogonal trajectories, population growth, and finding the current at a given time.

Unit II: Second order Linear Differential equations (15 Lectures)

1. Homogeneous and non-homogeneous second order linear differentiable equations: The space of solutions of the homogeneous equation as a vector space. Wronskian and linear independence of the solutions. The general solution of homogeneous differential equations. The general solution of a non-homogeneous second order equation. Complementary functions and particular integrals.

2. The homogeneous equation with constant coefficients. auxiliary equation. The general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation.

3. Non-homogeneous equations: The method of undetermined coefficients. The method of variation of parameters.

Unit III: Linear System of ODEs (15 Lectures)

Existence and uniqueness theorems to be stated clearly when needed in the sequel. Study of homogeneous linear system of ODEs in two variables: Let $a_1(t), a_2(t), b_1(t), b_2(t)$ be continuous real valued functions defined on $[a, b]$. Fix $t_0 \in [a, b]$. Then there exists a unique solution $x = x(t), y = y(t)$ valid throughout $[a, b]$ of the following system:

$$\frac{dx}{dt} = a_1(t)x + b_1(t)y,$$
$$\frac{dy}{dt} = a_2(t)x + b_2(t)y$$

satisfying the initial conditions $x(t_0) = x_0 \& y(t_0) = y_0$.
The Wronskian $W(t)$ of two solutions of a homogeneous linear system of ODEs in two variables, result: W (t) is identically zero or nowhere zero on [a, b]. Two linearly independent solutions and the general solution of a homogeneous linear system of ODEs in two variables.
Explicit solutions of Homogeneous linear systems with constant coefficients in two variables, examples.

Recommended Text Books for Unit I and II:

1. G. F. Simmons, Differential equations with applications and historical notes, McGraw Hill.

2. E. A. Coddington, An introduction to ordinary differential equations, Dover Books.

Recommended Text Book for Unit III:
G. F. Simmons, Differential equations with applications and historical notes, McGraw Hill.

USMT P04/UAMT P04 Practicals.

Suggested Practicals for USMT401/UAMT401:

1. Calculation of upper sum, lower sum and Riemann integral.
2. Problems on properties of Riemann integral.
3. Problems on fundamental theorem of calculus, mean value theorems, integration by parts, Leibnitz rule.
4. Convergence of improper integrals, applications of comparison tests, Abel's and Dirichlet's tests, and functions.
5. Beta Gamma Functions
6. Problems on area, volume, length.
7. Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals for USMT402/UAMT 402:

1. Examples and properties of groups.
2. Group of symmetry of equilateral triangle, rectangle, square.
3. Subgroups.
4. Cyclic groups, cyclic subgroups, finding generators of every subgroup of a cyclic group.
5. Left and right cosets of a subgroup, Lagrange's Theorem.

6. Group homomorphisms, isomorphisms.
7. Miscellaneous Theoretical questions based on full paper.

Suggested Practicals for USMT403:

1. Solving exact and non exact equations.
2. Linear and reducible to linear equations, applications to orthogonal trajectories, population growth, and finding the current at a given time.
3. Finding general solution of homogeneous and non-homogeneous equations, use of known solutions to find the general solution of homogeneous equations.
4. Solving equations using method of undetermined coefficients and method of variation of parameters.
5. Solving second order linear ODEs
6. Solving a system of first order linear ODES.
7. Miscellaneous Theoretical questions from all units.

Scheme of Examination

I. **Semester End Theory Examinations:** There will be a Semester-end external Theory examination of 100 marks for each of the courses USMT301/UAMT301, USMT302/UAMT302, USMT303 of Semester III and USMT401/UAMT401, USMT402/UAMT402, USMT403 of semester IV to be conducted by the University.

   1. Duration: The examinations shall be of 3 Hours duration.
   2. Theory Question Paper Pattern:
      a) There shall be FIVE questions. The first question Q1 shall be of objective type for 20 marks based on the entire syllabus. The next three questions Q2, Q2, Q3 shall be of 20 marks, each based on the units I, II, III respectively. The fifth question Q5 shall be of 20 marks based on the entire syllabus.
      b) All the questions shall be compulsory. The questions Q2, Q3, Q4, Q5 shall have internal choices within the questions. Including the choices, the marks for each question shall be 30-32.
      c) The questions Q2, Q3, Q4, Q5 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.
      d) The question Q1 may be subdivided into 10 sub-questions of 2 marks each.

II. **Semester End Examinations Practicals:**
At the end of the Semesters III and IV, Practical examinations of three hours duration and 150 marks shall be conducted for the courses USMTP03, USMTP04.

At the end of the Semesters III and IV, Practical examinations of three hours duration and 150 marks shall be conducted for the courses UAMTP03, UAMTP04.

In semester III, the Practical examinations for USMT301/UAMT301 and USMT302/UAMT302 are held together by the college. The Practical examination for USMT303 is held **separately** by the college.

In semester IV, the Practical examinations for USMT401/UAMT401 and USMT402/UAMT402 are held together by the college. The Practical examination for USMT403 is held **separately** by the college.

Paper pattern: The question paper shall have three parts A, B, C.
Each part shall have two Sections.

Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions. ($8 \times 3 = 24$ Marks)

Section II Problems: Attempt any Two out of Three. ($8 \times 2 = 16$ Marks)

Practical Course	Part A	Part B	**Part C**	Marks out of	duration
USMTP03	Questions from USMT301	Questions from USMT302	Questions from USMT303	120	3 hours
UAMTP03	Questions from UAMT301	Questions from UAMT302	—	80	2 hours
USMTP04	Questions from USMT401	Questions from USMT402	Questions from USMT403	120	3 hours
UAMTP03	Questions from UAMT401	Questions from UAMT402	—	80	2 hours

Marks for Journals and Viva:
For each course USMT301/UAMT301, USMT302/UAMT302, USMT303, USMT401/UAMT401, USMT402/UAMT402 and USMT403:

1. Journals: 5 marks.
2. Viva: 5 marks.

Each Practical of every course of Semester III and IV shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal. A student must have a certified journal before appearing for the practical examination.