

(UNIVERSITY OF MUMBAI)

Syllabus for: S.Y.B.Sc./S.Y.B.A.

Program: B.Sc./B/A.

Course: Mathematics

Choice based Credit System (CBCS)

with effect from the
academic year 2018-19

SEMESTER III

CALCULUS III				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 301, UAMT 301	I	Functions of several variables	2	3
	II	Differentiation		
	III	Applications		
ALGEBRA III				
USMT 302 ,UAMT 302	I	Linear Transformations and Matrices	2	3
	II	Determinants		
	III	Inner Product Spaces		
DISCRETE MATHEMATICS				
USMT 303	I	Permutations and Recurrence Relation	2	3
	II	Preliminary Counting		
	III	Advanced Counting		
PRACTICALS				
USMTP03		Practicals based on USMT301, USMT 302 and USMT 303	3	5
UAMTP03		Practicals based on UAMT301, UAMT 302	2	4

SEMESTER IV

CALCULUS IV				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 401, UAMT 401	I	Riemann Integration	2	3
	II	Indefinite Integrals and Improper Integrals		
	III	Beta and Gamma Functions And Applications		
ALGEBRA IV				
USMT 402 ,UAMT 402	I	Groups and Subgroups	2	3
	II	Cyclic Groups and Cyclic subgroups		
	III	Lagrange's Theorem and Group Homomorphism		
ORDINARY DIFFERENTIAL EQUATIONS				
USMT 403	I	First order First degree Differential equations	2	3
	II	Second order Linear Differential equations		
	III	Linear System of Ordinary Differential Equations		
PRACTICALS				
USMTP04		Practicals based on USMT401, USMT 402 and USMT 403	3	5
UAMTP04		Practicals based on UAMT401, UAMT 402	2	4

2. Gradient of a scalar field, geometric properties of gradient, level sets and tangent planes.
3. Chain rule for scalar fields.
4. Higher order partial derivatives, mixed partial derivatives, sufficient condition for equality of mixed partial derivative.

Reference for Unit II:

Sections 8.11, 8.12, 8.13, 8.14, 8.15, 8.16, 8.17, 8.23 of Calculus, Vol.2 (Second Edition) by T. Apostol, John Wiley.

Unit III: Applications (15 lectures)

1. Second order Taylor's formula for scalar fields.
2. Differentiability of vector fields, definition of differentiability of a vector field at a point, Jacobian matrix, differentiability of a vector field at a point implies continuity. The chain rule for derivative of vector fields (statements only)
3. Mean value inequality.
4. Hessian matrix, Maxima, minima and saddle points.
5. Second derivative test for extrema of functions of two variables.
6. Method of Lagrange Multipliers.

Reference for Unit III:

Sections 8.18, 8.19, 8.20, 8.21, 8.22, 9.9, 9.10, 9.11, 9.12, 9.13, 9.14 9.13, 9.14 from Apostol, Calculus Vol. 2, (Second Edition) by T. Apostol.

Recommended Text Books:

1. T. Apostol: Calculus, Vol. 2, John Wiley.
2. J. Stewart, Calculus, Brooke/ Cole Publishing Co.

Additional Reference Books

- (1) G.B. Thoman and R. L. Finney, Calculus and Analytic Geometry, Ninth Edition, Addison-Wesley, 1998.
- (2) Sudhir R. Ghorpade and Balmohan V. Limaye, A Course in Multivariable Calculus and Analysis, Springer International Edition.
- (3) Howard Anton, Calculus- A new Horizon, Sixth Edition, John Wiley and Sons Inc, 1999.

USMT 302/UAMT 302: ALGEBRA III

Note: Revision of relevant concepts is necessary.

Unit 1: Linear Transformations and Matrices (15 lectures)

1. Review of linear transformations: Kernel and image of a linear transformation, Rank-Nullity theorem (with proof), Linear isomorphisms, inverse of a linear isomorphism, Any n -dimensional real vector space is isomorphic to \mathbb{R}^n .
2. The matrix units, row operations, elementary matrices, elementary matrices are invertible and an invertible matrix is a product of elementary matrices.
3. Row space, column space of an $m \times n$ matrix, row rank and column rank of a matrix, Equivalence of the row and the column rank, Invariance of rank upon elementary row or column operations.
4. Equivalence of rank of an $m \times n$ matrix A and rank of the linear transformation $L_A : \mathbb{R}^n \rightarrow \mathbb{R}^m$ ($L_A(X) = AX$). The dimension of solution space of the system of linear equations $AX = 0$ equals $n - \text{rank}(A)$.
5. The solutions of non-homogeneous systems of linear equations represented by $AX = B$, Existence of a solution when $\text{rank}(A) = \text{rank}(A, B)$, The general solution of the system is the sum of a particular solution of the system and the solution of the associated homogeneous system.

Reference for Unit 1: Chapter VIII, Sections 1, 2 of Introduction to Linear Algebra, Serge Lang, Springer Verlag and Chapter 4, of Linear Algebra A Geometric Approach, S. Kumaresan, Prentice-Hall of India Private Limited, New Delhi.

Unit II: Determinants (15 Lectures)

1. Definition of determinant as an n -linear skew-symmetric function from $\mathbb{R}^n \times \mathbb{R}^n \times \dots \times \mathbb{R}^n \rightarrow \mathbb{R}$ such that determinant of (E^1, E^2, \dots, E^n) is 1, where E^j denotes the j^{th} column of the $n \times n$ identity matrix I_n . Determinant of a matrix as determinant of its column vectors (or row vectors). Determinant as area and volume.
2. Existence and uniqueness of determinant function via permutations, Computation of determinant of $2 \times 2, 3 \times 3$ matrices, diagonal matrices, Basic results on determinants such as $\det(A^t) = \det(A)$, $\det(AB) = \det(A)\det(B)$, Laplace expansion of a determinant, Vandermonde determinant, determinant of upper triangular and lower triangular matrices.
3. Linear dependence and independence of vectors in \mathbb{R}^n using determinants, The existence and uniqueness of the system $AX = B$, where A is an $n \times n$ matrix with $\det(A) \neq 0$, Co-factors and minors, Adjoint of an $n \times n$ matrix A , Basic results such as $A \text{adj}(A) = \det(A)I_n$. An $n \times n$ real matrix A is invertible if and only if $\det(A) \neq 0$, $A^{-1} = \frac{1}{\det(A)} \text{adj}(A)$ for an invertible matrix A , Cramer's rule.
4. Determinant as area and volume.

References for Unit 2: Chapter VI of Linear Algebra A geometric approach, S. Kumaresan, Prentice Hall of India Private Limited, 2001 and Chapter VII Introduction to Linear Algebra, Serge Lang, Springer Verlag.

Unit III: Inner Product Spaces (15 Lectures)

1. Dot product in \mathbb{R}^n , Definition of general inner product on a vector space over \mathbb{R} . Examples of inner product including the inner product $\langle f, g \rangle = \int_{-\pi}^{\pi} f(t)g(t) dt$ on $C[-\pi, \pi]$, the space of continuous real valued functions on $[-\pi, \pi]$.

2. Norm of a vector in an inner product space. Cauchy-Schwartz inequality, Triangle inequality, Orthogonality of vectors, Pythagoras theorem and geometric applications in \mathbb{R}^2 , Projections on a line, The projection being the closest approximation, Orthogonal complements of a subspace, Orthogonal complements in \mathbb{R}^2 and \mathbb{R}^3 . Orthogonal sets and orthonormal sets in an inner product space, Orthogonal and orthonormal bases. Gram-Schmidt orthogonalization process, Simple examples in $\mathbb{R}^3, \mathbb{R}^4$.

Reference of Unit 3: Chapter VI, Sections 1,2 of Introduction to Linear Algebra, Serge Lang, Springer Verlag and Chapter 5, of Linear Algebra A Geometric Approach, S. Kumaresan, Prentice-Hall of India Private Limited, New Delhi.

Recommended Books:

1. Serge Lang: Introduction to Linear Algebra, Springer Verlag.
2. S. Kumaresan: Linear Algebra A geometric approach, Prentice Hall of India Private Limited.

Additional Reference Books:

1. M. Artin: Algebra, Prentice Hall of India Private Limited.
2. K. Hoffman and R. Kunze: Linear Algebra, Tata McGraw-Hill, New Delhi.
3. Gilbert Strang: Linear Algebra and its applications, International Student Edition.
4. L. Smith: Linear Algebra, Springer Verlag.
5. A. Ramachandra Rao and P. Bhima Sankaran: Linear Algebra, Tata McGraw-Hill, New Delhi.
6. T. Banchoff and J. Wermer: Linear Algebra through Geometry, Springer Verlag Newyork, 1984.
7. Sheldon Axler: Linear Algebra done right, Springer Verlag, Newyork.
8. Klaus Janich: Linear Algebra.
9. Otto Bretcher: Linear Algebra with Applications, Pearson Education.
10. Gareth Williams: Linear Algebra with Applications, Narosa Publication.

USMT 303: Discrete Mathematics

Unit I: Permutations and Recurrence relation (15 lectures)

1. Permutation of objects, S_n , composition of permutations, results such as every permutation is a product of disjoint cycles, every cycle is a product of transpositions, even and odd permutation, rank and signature of a permutation, cardinality of S_n, A_n
2. Recurrence Relations, definition of non-homogeneous, non-homogeneous, linear, non-linear recurrence relation, obtaining recurrence relation in counting problems, solving homogeneous as well as non homogeneous recurrence relations by using iterative methods, solving a homogeneous recurrence relation of second degree using algebraic method proving the necessary result.

- (3) Ajit Kumar, S.Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
 (4) T. Apostol, Calculus Vol.2, John Wiley.
 (5) K. Stewart, Calculus, Booke/Cole Publishing Co, 1994.
 (6) J. E. Marsden, A.J. Tromba and A. Weinstein, Basic multivariable calculus.
 (7) Bartle and Sherbet, Real analysis.

USMT 402/ UAMT 402: ALGEBRA IV

Unit I: Groups and Subgroups (15 Lectures)

- (a) Definition of a group, abelian group, order of a group, finite and infinite groups. Examples of groups including:
- i) $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$ under addition.
 - ii) $\mathbb{Q}^*(= \mathbb{Q} \setminus \{0\}), \mathbb{R}^*(= \mathbb{R} \setminus \{0\}), \mathbb{C}^*(= \mathbb{C} \setminus \{0\}), \mathbb{Q}^+$ (= positive rational numbers) under multiplication.
 - iii) \mathbb{Z}_n , the set of residue classes modulo n under addition.
 - iv) $U(n)$, the group of prime residue classes modulo n under multiplication.
 - v) The symmetric group S_n .
 - vi) The group of symmetries of a plane figure. The Dihedral group D_n as the group of symmetries of a regular polygon of n sides (for $n = 3, 4$).
 - vii) Klein 4-group.
 - viii) Matrix groups $M_{n \times n}(\mathbb{R})$ under addition of matrices, $GL_n(\mathbb{R})$, the set of invertible real matrices, under multiplication of matrices.
 - ix) Examples such as S^1 as subgroup of C , μ_n the subgroup of n -th roots of unity.
- (b) Properties such as
- 1) In a group (G, \cdot) the following indices rules are true for all integers n, m .
 - i) $a^n a^m = a^{n+m}$ for all a in G .
 - ii) $(a^n)^m = a^{nm}$ for all a in G .
 - iii) $(ab)^n = a^n b^n$ for all ab in G whenever $ab = ba$.
 - 2) In a group (G, \cdot) the following are true:
 - i) The identity element e of G is unique.
 - ii) The inverse of every element in G is unique.
 - iii) $(a^{-1})^{-1} = a$ for all a in G .
 - iv) $(a.b)^{-1} = b^{-1}a^{-1}$ for all a, b in G .
 - v) If $a^2 = e$ for every a in G then (G, \cdot) is an abelian group.
 - vi) $(aba^{-1})^n = ab^n a^{-1}$ for every a, b in G and for every integer n .
 - vii) If $(a.b)^2 = a^2.b^2$ for every a, b in G then (G, \cdot) is an abelian group.
 - viii) (\mathbb{Z}_n^*, \cdot) is a group if and only if n is a prime.
 - 3) Properties of order of an element such as: (n and m are integers.)
 - i) If $o(a) = n$ then $a^m = e$ if and only if n/m .
 - ii) If $o(a) = nm$ then $o(a^n) = m$.
 - iii) If $o(a) = n$ then $o(a^m) = \frac{n}{(n, m)}$, where (n, m) is the GCD of n and m .

- iv) $o(aba^{-1}) = o(b)$ and $o(ab) = o(ba)$.
- v) If $o(a) = m$ and $o(b) = n, ab = ba, (n, m) = 1$ then $o(ab) = nm$.

(c) Subgroups

- i) Definition, necessary and sufficient condition for a non-empty set to be a Subgroup.
- ii) The center $Z(G)$ of a group is a subgroup.
- iii) Intersection of two (or a family of) subgroups is a subgroup.
- iv) Union of two subgroups is not a subgroup in general. Union of two subgroups is a subgroup if and only if one is contained in the other.
- v) If H and K are subgroups of a group G then HK is a subgroup of G if and only if $HK = KH$.

Reference for Unit I:

- (1) I.N. Herstein, Topics in Algebra.
- (2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Unit II: Cyclic groups and cyclic subgroups (15 Lectures)

- (a) Cyclic subgroup of a group, cyclic groups, (examples including \mathbb{Z}, \mathbb{Z}_n and μ_n).
- (b) Properties such as:
 - (i) Every cyclic group is abelian.
 - (ii) Finite cyclic groups, infinite cyclic groups and their generators.
 - (iii) A finite cyclic group has a unique subgroup for each divisor of the order of the group.
 - (iv) Subgroup of a cyclic group is cyclic.
 - (v) In a finite group $G, G = \langle a \rangle$ if and only if $o(G) = o(a)$.
 - (vi) If $G = \langle a \rangle$ and $o(a) = n$ then $G = \langle a^m \rangle$ if and only if $(n, m) = 1$.
 - (vii) If G is a cyclic group of order p^n and $H < G, K < G$ then prove that either $H \subseteq K$ or $K \subseteq H$.

References for Unit II:

- (1) I.N. Herstein, Topics in Algebra.
- (2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Unit III: Lagrange's Theorem and Group homomorphism (15 Lectures)

- (a) Definition of Coset and properties such as :
 - 1) IF H is a subgroup of a group G and $x \in G$ then
 - (i) $xH = H$ if and only if $x \in H$.
 - (ii) $Hx = H$ if and only if $x \in H$.
 - 2) If H is a subgroup of a group G and $x, y \in G$ then
 - (i) $xH = yH$ if and only if $x^{-1}y \in H$.
 - (ii) $Hx = Hy$ if and only if $xy^{-1} \in H$.

- 3) Lagrange's theorem and consequences such as Fermat's Little theorem, Euler's theorem and if a group G has no nontrivial subgroups then order of G is a prime and G is Cyclic.
- (b) Group homomorphisms and isomorphisms, automorphisms
- i) Definition.
 - ii) Kernel and image of a group homomorphism.
 - iii) Examples including inner automorphism.

Properties such as:

- (1) $f : G \longrightarrow G'$ is a group homomorphism then $\ker f < G$.
- (2) $f : G \longrightarrow G'$ is a group homomorphism then $\ker f = \{e\}$ if and only if f is 1-1.
- (3) $f : G \longrightarrow G'$ is a group homomorphism then
 - (i) G is abelian if and only if G' is abelian.
 - (ii) G is cyclic if and only if G' is cyclic.

Reference for Unit III:

1. I.N. Herstein, Topics in Algebra.
2. P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Recommended Books:

1. I.N. Herstein, Topics in Algebra, Wiley Eastern Limited, Second edition.
2. N.S. Gopalkrishnan, University Algebra, Wiley Eastern Limited.
3. M. Artin, Algebra, Prentice Hall of India, New Delhi.
4. P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.
5. J.B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.
6. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.
7. COmbinatioial Techniques by Sharad S. Sane, Hindustan Book Agency.

Additional Reference Books:

1. S. Adhikari. An introduction to Commutative Algebra and Number theory. Narosa Publishing House.
2. T. W. Hungerford. Algebra, Springer.
3. D. Dummit, R. Foote. Abstract Algebra, John Wiley & Sons, Inc.
4. I.S. Luther, I.B.S. Passi. Algebra. Vol. I and II.

6. Group homomorphisms, isomorphisms.
7. Miscellaneous Theoretical questions based on full paper.

Suggested Practicals for USMT403:

1. Solving exact and non exact equations.
2. Linear and reducible to linear equations, applications to orthogonal trajectories, population growth, and finding the current at a given time.
3. Finding general solution of homogeneous and non-homogeneous equations, use of known solutions to find the general solution of homogeneous equations.
4. Solving equations using method of undetermined coefficients and method of variation of parameters.
5. Solving second order linear ODEs
6. Solving a system of first order linear ODES.
7. Miscellaneous Theoretical questions from all units.

Scheme of Examination

I. **Semester End Theory Examinations:** There will be a Semester-end external Theory examination of 100 marks for each of the courses USMT301/UAMT301, USMT302/UAMT302, USMT303 of Semester III and USMT401/UAMT401, USMT402/UAMT402, USMT403 of semester IV to be conducted by the University.

1. Duration: The examinations shall be of 3 Hours duration.
2. Theory Question Paper Pattern:
 - a) There shall be FIVE questions. The first question Q1 shall be of objective type for 20 marks based on the entire syllabus. The next three questions Q2, Q2, Q3 shall be of 20 marks, each based on the units I, II, III respectively. The fifth question Q5 shall be of 20 marks based on the entire syllabus.
 - b) All the questions shall be compulsory. The questions Q2, Q3, Q4, Q5 shall have internal choices within the questions. Including the choices, the marks for each question shall be 30-32.
 - c) The questions Q2, Q3, Q4, Q5 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.
 - d) The question Q1 may be subdivided into 10 sub-questions of 2 marks each.

II. **Semester End Examinations Practicals:**

At the end of the Semesters III and IV, Practical examinations of three hours duration and 150 marks shall be conducted for the courses USMTP03, USMTP04.

At the end of the Semesters III and IV, Practical examinations of three hours duration and 150 marks shall be conducted for the courses UAMTP03, UAMTP04.

In semester III, the Practical examinations for USMT301/UAMT301 and USMT302/UAMT302 are held together by the college. The Practical examination for USMT303 is held **separately** by the college.

In semester IV, the Practical examinations for USMT401/UAMT401 and USMT402/UAMT402 are held together by the college. The Practical examination for USMT403 is held **separately** by the college.

Paper pattern: The question paper shall have three parts A, B, C. Each part shall have two Sections.

Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions. ($8 \times 3 = 24$ Marks)

Section II Problems: Attempt any Two out of Three. ($8 \times 2 = 16$ Marks)

Practical Course	Part A	Part B	Part C	Marks out of	duration
USMTP03	Questions from USMT301	Questions from USMT302	Questions from USMT303	120	3 hours
UAMTP03	Questions from UAMT301	Questions from UAMT302	—	80	2 hours
USMTP04	Questions from USMT401	Questions from USMT402	Questions from USMT403	120	3 hours
UAMTP03	Questions from UAMT401	Questions from UAMT402	—	80	2 hours

Marks for Journals and Viva:

For each course USMT301/UAMT301, USMT302/UAMT302, USMT303, USMT401/UAMT401, USMT402/UAMT402 and USMT403:

1. Journals: 5 marks.
2. Viva: 5 marks.

Each Practical of every course of Semester III and IV shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal. A student must have a certified journal before appearing for the practical examination.