UNIVERSITY OF MUMBAI

Syllabus

for F. Y. B. Sc. / F. Y. B. A. Semester I & II (CBCS)

Program: B. Sc. / B. A.

Course: Mathematics

with effect from the academic year 2020-2021

F. Y. B. Sc. (CBCS) SEMESTER I

CALCULUS I				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 101	I	Real Number System	2	3
	II	Sequences in $\mathbb{R}$		
	III	First Order First Degree Differential Equations		
ALGEBRA I				
USMT 102	I	Integers and Divisibility	2	3
	II	Functions, Relations and Binary Operations		
	III	Polynomials		
PRACTICALS				
USMTP01	-	Practicals based on USMT101, USMT102	2	2

F. Y. B. A. (CBCS) SEMESTER I

CALCULUS I				
Course Code	UNIT	TOPICS	Credits	L/Week
UAMT 101	I	Real Number System	3	3
	II	Real Sequences		
	III	First Order First Degree Differential Equations		
Tutorials				
	-	Tutorials based on UAMT101		

F. Y. B. Sc. (CBCS) SEMESTER II

CALCULUS II				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 201	I	Limits and Continuity	2	3
	II	Differentiability of functions		
	III	Applications of Differentiability		
DISCRETE MATHEMATICS				
USMT 202	I	Preliminary Counting	2	3
	II	Advanced Counting		
	III	Permutations and Recurrence Relation		
PRACTICALS				
USMTP02	-	Practicals based on USMT201, USMT202	2	2

F. Y. B. A. (CBCS) SEMESTER II

CALCULUS II				
Course Code	UNIT	TOPICS	Credits	L/Week
	I	Limits and Continuity		
	II	Differentiability of functions		
UAMT 201	III	Applications of Differentiability	3	3
TUTORIALS				
	-	Tutorials based on UAMT201		

Revised Syllabus in Mathematics
Choice Based Credit System
F. Y. B. Sc. / B. A. 2020-2021

Preamble:
The University of Mumbai has brought into force the revised syllabi as per the Choice Based Credit System (CBCS) for the First year B. Sc/ B. A. Programme in Mathematics from the academic year 2020-2021.

Mathematics has been fundamental to the development of science and technology. In recent decades, the extent of application of Mathematics to real world problems has increased by leaps and bounds. Taking into consideration the rapid changes in science and technology and new approaches in different areas of mathematics and related subjects like Physics, Statistics and Computer Sciences, the board of studies in Mathematics with concern of teachers of Mathematics from different colleges affiliated to University of Mumbai has prepared the syllabus of F.Y.B. Sc. / F. Y. B. A. Mathematics. The present syllabi of F. Y. B. Sc. for Semester I and Semester II has been designed as per U. G. C. Model curriculum so that the students learn Mathematics needed for these branches, learn basic concepts of Mathematics and are exposed to rigorous methods gently and slowly. The syllabi of F. Y. B. Sc. / F. Y. B. A. would consist of two semesters and each semester would comprise of two courses for F. Y. B. Sc. Mathematics and one course for each semester for F. Y. B. A. Mathematics. Course I is 'Calculus I and Calculus II'. Calculus is applied and needed in every conceivable branch of science. Course II, 'Algebra I and Discrete Mathematics' develops mathematical reasoning and logical thinking and has applications in science and technology.

Aims:

(1) Give the students a sufficient knowledge of fundamental principles, methods and a clear perception of innumerous power of mathematical ideas and tools and know how to use them by modeling, solving and interpreting.

(2) Reflecting the broad nature of the subject and developing mathematical tools for continuing further study in various fields of science.

(3) Enhancing students' overall development and to equip them with mathematical modeling abilities, problem solving skills, creative talent and power of communication necessary for various kinds of employment.

(4) A student should get adequate exposure to global and local concerns that explore them many aspects of Mathematical Sciences

Course outcomes:

1. Calculus (Sem I & II): This course gives introduction to basic concepts of Analysis with rigor and prepares students to study further courses in Analysis. Formal proofs are given lot of emphasis in this course which also enhances understanding of the subject of Mathematics as a whole. The portion on first order, first degree differentials prepares learner to get solutions of so many kinds of problems in all subjects of Science and also prepares learner for further studies of differential equations and related fields.

2. Algebra I (Sem I) & Discrete Mathematics (Sem II): This course gives expositions to number systems (Natural Numbers & Integers), like divisibility and prime numbers and

their properties. These topics later find use in advanced subjects like cryptography and its uses in cyber security and such related fields.

Teaching Pattern for Semester I

[1.] Three lectures per week per course.

[2.] One Practical per week per batch for each of the courses USMT101, USMT 102 (the batches to be formed as prescribed by the University).

[3.] One Tutorial per week per batch for course UAMT101 (the batches to be formed as prescribed by the University).

Teaching Pattern for Semester II

[1.] Three lectures per week per course.

[2.] One Practical per week per batch for each of the courses USMT201, USMT 202. (the batches to be formed as prescribed by the University).

[3.] One Tutorial per week per batch for the course UAMT201 (the batches to be formed as prescribed by the University).

ALGEBRA I
USMT 102

Prerequisite :
Set Theory: Set, subset, union and intersection of two sets, empty set, universal set, complement of a set, De Morgan's laws, Cartesian product of two sets, Relations, Permutations ${}^nP_r$ and Combinations ${}^nC_r$.
Complex numbers: Addition and multiplication of complex numbers, modulus, amplitude and conjugate of a complex number.

Unit I : Integers & Divisibility (15 Lectures)

(1) Statements of well-ordering property of non-negative integers, Principle of finite induction (first and second) as a consequence of Well-Ordering Principle.

(2) Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m.) of two non zero integers, basic properties of g.c.d. such as existence and uniqueness of g.c.d. of two non zero integers $a\&b$ and that the g.c.d. can be expressed as $ma + nb$ for some $m, n \in \mathbb{Z}$, Euclidean algorithm.

(3) Primes, Euclid's lemma, Fundamental Theorem of arithmetic, The set of primes is infinite, there are arbitrarily large gaps between primes, there exists infinitely many primes of the form $4n - 1$ or of the form $6n - 1$.

(4) Congruence, definition and elementary properties, Results about linear congruence equations. Examples.

Unit II : Functions, Relations and Binary Operations (15 Lectures)

(1) Definition of relation and function, domain, co-domain and range of a function, composite functions, examples, Direct image $f(A)$ and inverse image $f^{-1}(B)$ for a function f, injective, surjective, bijective functions, Composite of injective, surjective, bijective functions when defined, invertible functions, bijective functions are invertible and conversely, examples of functions including constant, identity, projection, inclusion, Binary operation as a function, properties, examples.

(2) Equivalence relation, Equivalence classes, properties such as two equivalences classes are either identical or disjoint, Definition of partition, every partition gives an equivalence relation and vice versa.

(3) Congruence is an equivalence relation on $\mathbb{Z}$, Residue classes and partition of $\mathbb{Z}$, Addition modulo$n$, Multiplication modulo n, examples.

Unit III: Polynomials (15 Lectures)

(1) Definition of a polynomial, polynomials over F where $F = \mathbb{Q}, \mathbb{R}$ or $\mathbb{C}$, Algebra of polynomials, degree of polynomial, basic properties.

(2) Division algorithm in $F[X]$ (without proof), and g.c.d of two polynomials and its basic properties, Euclidean algorithm (proof of the above results may be given only in the case of $\mathbb{Q}[X]$ with a remark that the results as well as the proofs remain valid in the case of $\mathbb{R}[X]$ or $\mathbb{C}[X]$).

(3) Roots of a polynomial, relation between roots and coefficients, multiplicity of a root. Elementary consequences such as the following.

(i) Remainder theorem, Factor theorem.

(ii) A polynomial of degree n has at most n roots.

(iii) Complex and non-real roots of a polynomials in $\mathbb{R}[X]$ occur in conjugate pairs.

(Emphasis on examples and problems in polynomials with real coefficients).

(4) Necessary condition for a rational number $\frac{p}{q}$ to be a root of a polynomial with integer coefficients (viz. p divides the constant coefficient and q divides the leading coefficient), corollary for monic polynomials (viz. a rational root of monic polynomial with integer coefficients is necessarily an integer). Simple consequence such as the irrationality is necessarily of $\sqrt{p}$ for any prime number p. Irreducible polynomials in $\mathbb{Q}[x]$, Unique Factorisation Theorem. Examples.

Reference Books:

1. David M. Burton, Elementary Number Theory, Seventh Edition, McGraw Hill Education (India) Private Ltd.

2. 2. Norman L. Biggs, Discrete Mathematics, Revised Edition, Clarendon Press, Oxford 1989.

Additional Reference Books

1. I. Niven and S. Zuckerman, Introduction to the theory of numbers, Third Edition, Wiley Eastern, New Delhi, 1972.

2. G. Birkoff and S. Maclane, A Survey of Modern Algebra, Third Edition, Mac Millan, New York, 1965.

3. N. S. Gopalkrishnan, University Algebra, Ne Age International Ltd, Reprint 2013.

4. I .N. Herstein, Topics in Algebra, John Wiley, 2006.

5. P. B. Bhattacharya S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, New Age International, 1994.

6. Kenneth Rosen, Discrete Mathematics and its applications, Mc-Graw Hill, International Edition, Mathematics Series.

4. Sudhir Ghorpade and Balmohan Limaye, A course in Calculus and Real Analysis, Springer International Ltd, 2000.

Additional Reference:

1. Richard Courant and Fritz John, A Introduction to Calculus and Analysis, Volume-I, Springer.

2. Ajit Kumar and S. Kumaresan, A Basic course in Real Analysis, CRC Press, 2014.

3. K. G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.

4. G. B. Thomas, Calculus, 12th Edition 2009

USMT 202: DISCRETE MATHEMATICS

Unit I: Preliminary Counting (15 Lectures)

(1) Finite and infinite sets, countable and uncountable sets examples such as $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{N} \times \mathbb{N}$, $\mathbb{Q}$ $(0,1)$, $\mathbb{R}$.

(2) Addition and multiplication Principle, counting sets of pairs, two ways counting.

(3) Stirling numbers of second kind. Simple recursion formulae satisfied by $S(n,k)$ for $k = 1, 2, \cdots, n-1, n$.

(4) Pigeonhole principle simple and strong form and examples, its applications to geometry.

Unit II: Advanced Counting (15 Lectures)

(1) Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems.

(2) Binomial and Multinomial Theorem, Pascal identity, examples of standard identities such as the following with emphasis on combinatorial proofs.

- $\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$
- $\sum_{i=0}^{k} \binom{k}{i}^2 = \binom{2k}{k}$
- $\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$
- $\sum_{i=0}^{n} \binom{n}{i} = 2^n$

(3) Non-negative integer solutions of equation $x_1 + x_2 + \cdots + x_k = n$.

(4) Principal of inclusion and exclusion, its applications, derangements, explicit formula for d_n, deriving formula for Euler's function $\phi(n)$.

Unit III: Permutations and Recurrence relation (15 lectures)

(1) Permutation of objects, S_n, composition of permutations, results such as every permutation is a product of disjoint cycles, every cycle is a product of transpositions, signature of a permutation, even and odd permutations, cardinality of S_n, A_n.

(2) Recurrence Relations, definition of homogeneous, non-homogeneous, linear, non-linear recurrence relation, obtaining recurrence relations of Tower of Hanoi, Fibonacci sequence, etc. in counting problems, solving homogeneous as well as non homogeneous recurrence relations by using iterative methods, solving a homogeneous recurrence relation of second degree using algebraic method proving the necessary result.

Recommended Books:

1. Norman Biggs, Discrete Mathematics, Oxford University Press.
2. Richard Brualdi, Introductory Combinatorics, John Wiley and sons.
3. V. Krishnamurthy, Combinatorics-Theory and Applications, Affiliated East West Press.
4. Discrete Mathematics and its Applications, Tata McGraw Hills.
5. Schaum's outline series, Discrete mathematics,
6. Allen Tucker, Applied Combinatorics, John Wiley and Sons.
7. Sharad Sane, Combinatorial Techniques, Springer.

PRACTICALS FOR F.Y.B.Sc
USMTP02-Practicals

A. **Practicals for USMT201 :**

(1) Limit of a function and Sandwich theorem, Continuous and discontinuous function.

(2) Algebra of limits and continuous functions, Intermediate Value theorem, Bolzano-Weierstrass theorem.

(3) Properties of differentiable functions, derivatives of inverse functions and implicit functions.

(4) Higher order derivatives, Leibnitz Rule.

(5) Mean value theorems and its applications, L'Hospital's Rule, Increasing and Decreasing functions.

(6) Extreme values, Taylor's Theorem and Curve Sketching.

(7) Miscellaneous Theoretical Questions based on full paper.

B. **Practicals for USMT202:**

(1) Counting principles, Two way counting.

(2) Stirling numbers of second kind, Pigeon hole principle.

(3) Multinomial theorem, identities, permutation and combination of multi-set.

(4) Inclusion-Exclusion principle. Euler phi function.

(5) Composition of permutations, signature of permutation, inverse of permutation.

(6) Recurrence relation.

(7) Miscellaneous Theoretical Questions based on full paper.