UNIVERSITY OF MUMBAI

Syllabus

for F. Y. B. Sc. / F. Y. B. A. Semester I & II (CBCS)

Program: B. Sc. / B. A.

Course: Mathematics

with effect from the academic year 2020-2021

F. Y. B. Sc. (CBCS) SEMESTER I

CALCULUS I				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 101	I	Real Number System	2	3
	II	Sequences in $\mathbb{R}$		
	III	First Order First Degree Differential Equations		
ALGEBRA I				
USMT 102	I	Integers and Divisibility	2	3
	II	Functions, Relations and Binary Operations		
	III	Polynomials		
PRACTICALS				
USMTP01	-	Practicals based on USMT101, USMT102	2	2

F. Y. B. A. (CBCS) SEMESTER I

CALCULUS I				
Course Code	UNIT	TOPICS	Credits	L/Week
UAMT 101	I	Real Number System	3	3
	II	Real Sequences		
	III	First Order First Degree Differential Equations		
Tutorials				
	-	Tutorials based on UAMT101		

F. Y. B. Sc. (CBCS) SEMESTER II

CALCULUS II				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 201	I	Limits and Continuity	2	3
	II	Differentiability of functions		
	III	Applications of Differentiability		
DISCRETE MATHEMATICS				
USMT 202	I	Preliminary Counting	2	3
	II	Advanced Counting		
	III	Permutations and Recurrence Relation		
PRACTICALS				
USMTP02	-	Practicals based on USMT201, USMT202	2	2

F. Y. B. A. (CBCS) SEMESTER II

CALCULUS II				
Course Code	UNIT	TOPICS	Credits	L/Week
	I	Limits and Continuity		
	II	Differentiability of functions		
UAMT 201	III	Applications of Differentiability	3	3
TUTORIALS				
	-	Tutorials based on UAMT201		

Revised Syllabus in Mathematics
Choice Based Credit System
F. Y. B. Sc. / B. A. 2020-2021

Preamble:

The University of Mumbai has brought into force the revised syllabi as per the Choice Based Credit System (CBCS) for the First year B. Sc/ B. A. Programme in Mathematics from the academic year 2020-2021.

Mathematics has been fundamental to the development of science and technology. In recent decades, the extent of application of Mathematics to real world problems has increased by leaps and bounds. Taking into consideration the rapid changes in science and technology and new approaches in different areas of mathematics and related subjects like Physics, Statistics and Computer Sciences, the board of studies in Mathematics with concern of teachers of Mathematics from different colleges affiliated to University of Mumbai has prepared the syllabus of F.Y.B. Sc. / F. Y. B. A. Mathematics. The present syllabi of F. Y. B. Sc. for Semester I and Semester II has been designed as per U. G. C. Model curriculum so that the students learn Mathematics needed for these branches, learn basic concepts of Mathematics and are exposed to rigorous methods gently and slowly. The syllabi of F. Y. B. Sc. / F. Y. B. A. would consist of two semesters and each semester would comprise of two courses for F. Y. B. Sc. Mathematics and one course for each semester for F. Y. B. A. Mathematics. Course I is 'Calculus I and Calculus II'. Calculus is applied and needed in every conceivable branch of science. Course II, 'Algebra I and Discrete Mathematics' develops mathematical reasoning and logical thinking and has applications in science and technology.

Aims:

(1) Give the students a sufficient knowledge of fundamental principles, methods and a clear perception of innumerous power of mathematical ideas and tools and know how to use them by modeling, solving and interpreting.

(2) Reflecting the broad nature of the subject and developing mathematical tools for continuing further study in various fields of science.

(3) Enhancing students' overall development and to equip them with mathematical modeling abilities, problem solving skills, creative talent and power of communication necessary for various kinds of employment.

(4) A student should get adequate exposure to global and local concerns that explore them many aspects of Mathematical Sciences

Course outcomes:

1. Calculus (Sem I & II): This course gives introduction to basic concepts of Analysis with rigor and prepares students to study further courses in Analysis. Formal proofs are given lot of emphasis in this course which also enhances understanding of the subject of Mathematics as a whole. The portion on first order, first degree differentials prepares learner to get solutions of so many kinds of problems in all subjects of Science and also prepares learner for further studies of differential equations and related fields.

2. Algebra I (Sem I) & Discrete Mathematics (Sem II): This course gives expositions to number systems (Natural Numbers & Integers), like divisibility and prime numbers and

their properties. These topics later find use in advanced subjects like cryptography and its uses in cyber security and such related fields.

Teaching Pattern for Semester I

[1.] Three lectures per week per course.

[2.] One Practical per week per batch for each of the courses USMT101, USMT 102 (the batches to be formed as prescribed by the University).

[3.] One Tutorial per week per batch for course UAMT101 (the batches to be formed as prescribed by the University).

Teaching Pattern for Semester II

[1.] Three lectures per week per course.

[2.] One Practical per week per batch for each of the courses USMT201, USMT 202. (the batches to be formed as prescribed by the University).

[3.] One Tutorial per week per batch for the course UAMT201 (the batches to be formed as prescribed by the University).

F.Y.B.Sc. / F.Y.B.A. Mathematics

SEMESTER I

USMT 101 / UAMT 101: CALCULUS I

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

Unit 1 : Real Number System (15 Lectures)

(1) Real number system $\mathbb{R}$ and order properties of $\mathbb{R}$, absolute value $|\ |$ and its properties.

(2) AM-GM inequality, Cauchy-Schwarz inequality, Intervals and neighbourhoods, interior points, limit point, Hausdorff property.

(3) Bounded sets, statements of l.u.b. axiom and its consequences, supremum and infimum, maximum and minimum, Archimedean property and its applications, density of rationals.

Unit II: Sequences in $\mathbb{R}$ (15 Lectures)

(1) Definition of a sequence and examples, Convergence of sequences, every convergent sequences is bounded. Limit of a convergent sequence and uniqueness of limit, Divergent sequences.

(2) Convergence of standard sequences like $\left(\frac{1}{1+na}\right)$ $\forall a > 0$, (b^n) $\forall b,\ 0 < b < 1$, $(c^{\frac{1}{n}})$ $\forall c > 0$, & $(n^{\frac{1}{n}})$.

(3) Algebra of convergent sequences, sandwich theorem, monotone sequences, monotone convergence theorem and consequences as convergence of $\left(\left(1+\frac{1}{n}\right)^n\right)$.

(4) Definition of subsequence, subsequence of a convergent sequence is convergent and converges to the same limit, definition of a Cauchy sequences, every convergent sequences s a Cauchy sequence and converse.

Unit III: First order First degree Differential equations (15 Lectures)

Review of Definition of a differential equation, order, degree, ordinary differential equation and partial differential equation, linear and non linear ODE. Solution of homogeneous and non-homogeneous differential equations of first order and first degree. Notion of partial derivatives.

(1) Exact Equations: General solution of Exact equations of first order and first degree. Necessary and sufficient condition for $Mdx + Ndy = 0$ to be exact. Non-exact equations: Rules for finding integrating factors (without proof) for non exact equations, such as :

   i) $\frac{1}{M\ x + N\ y}$ is an I.F. if $M\ x + N\ y \neq 0$ and $Mdx + Ndy = 0$ is homogeneous.

   ii) $\frac{1}{M\ x - N\ y}$ is an I.F. if $M\ x - N\ y \neq 0$ and $Mdx + Ndy = 0$ is of the form $f_1(x,y)\ y\ dx + f_2(x,y)\ x\ dy = 0$.

iii) $e^{\int f(x)\ dx}$ (resp $e^{\int g(y)\ dy}$) is an I.F. if $N \neq 0$ (resp $M \neq 0$) and $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ $\left(\text{resp}\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)\right)$ is a function of x (resp y) alone, say $f(x)$ (resp $g(y)$).

iv) Linear and reducible linear equations of first order, finding solutions of first order differential equations of the type for applications to orthogonal trajectories, population growth, and finding the current at a given time.

(2) Reduction of order :

(i) If the differential equation does not contain only the original function y, that is equations of Type $F(x, y', y'') = 0$.

(ii) If the differential equation does not contain the independent variable x that is, equations of Type $F(y, y', y'') = 0$.

Reference Books:

1. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. K. G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
3. R. G. Bartle- D. R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.
4. Sudhir Ghorpade and Balmohan Limaye, A course in Calculus and Real Analysis, Springer International Ltd, 2000.
5. G. F. Simmons, Differential Equations with Applications and Historical Notes, McGraw Hill, 1972.
6. E. A. Coddington , An Introduction to Ordinary Differential Equations.Prentice Hall, 1961.
7. W. E. Boyce, R. C. DiPrima, Elementary Differential Equations and Boundary Value Problems, Wiely, 2013.

Additional Reference Books

1. T. M. Apostol, Calculus Volume I, Wiley & Sons (Asia) Pte, Ltd.
2. Richard Courant-Fritz John, A Introduction to Calculus and Analysis, Volume I, Springer.
3. Ajit kumar and S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
4. James Stewart, Calculus, Third Edition, Brooks/ cole Publishing Company, 1994.
5. D. A. Murray, Introductory Course in Differential Equations, Longmans, Green and Co., 1897.
6. A. R. Forsyth, A Treatise on Differential Equations, MacMillan and Co.,1956.

Semester II
USMT 201 / UAMT201: CALCULUS II

Unit-I: Limits and Continuity (15 Lectures)

{Brief review: Domain and range of a function, injective function, surjective function, bijective function, composite of two functions (when defined), Inverse of a bijective function. Graphs of some standard functions such as $|x|$, e^x, $\log x$, ax^2+bx+c, $\frac{1}{x}$, x^n $n \geq 3)$, $\sin x$, $\cos x$, $\tan x$, $\sin\left(\frac{1}{x}\right)$, $x^2 \sin\left(\frac{1}{x}\right)$ over suitable intervals of $\mathbb{R}$. No direct questions to be added.}

(1) $\varepsilon - \delta$ definition of Limit of a function, uniqueness of limit if it exists, algebra of limits, limits of composite function, sandwich theorem, left-hand-limit $\lim_{x \longrightarrow a^-} f(x)$, right-hand-limit $\lim_{x \longrightarrow a^+} f(x)$, non-existence of limits, $\lim_{x \longrightarrow -\infty} f(x)$, $\lim_{x \longrightarrow \infty} f(x)$ and $\lim_{x \longrightarrow a} f(x) = \pm\infty$.

(2) Continuous functions: Continuity of a real valued function at a point and on a set using $\epsilon - \delta$ definition, examples, Continuity of a real valued function at end points of domain using $\epsilon - \delta$ definition, f is continuous at a if and only if $\lim_{x \longrightarrow a} f(x)$ exists and equals to $f(a)$, Sequential continuity, Algebra of continuous functions, discontinuous functions, examples of removable and essential discontinuity.

(3) Intermediate Value theorem and its applications, Bolzano-Weierstrass theorem (statement only): A continuous function on a closed and bounded interval is bounded and attains its bounds.

Unit-II: Differentiability of functions (15 Lectures)

(1) Differentiation of real valued function of one variable: Definition of differentiability of a function at a point of an open interval, examples of differentiable and non differentiable functions, differentiable functions are continuous but not conversely, algebra of differentiable functions.

(2) Chain rule, Higher order derivatives, Leibniz rule, Derivative of inverse functions, Implicit differentiation (only examples)

Unit-III: Applications of differentiability (15 Lectures)

(1) Rolle's Theorem, Lagrange's and Cauchy's Mean Value Theorems, applications and examples, Monotone increasing and decreasing functions, examples.

(2) L-Hospital rule (without proof), examples of indeterminate forms, Taylor's theorem with Lagrange's form of remainder with proof, Taylor polynomial and applications.

(3) Definition of critical point, local maximum/minimum, necessary condition, stationary points, second derivative test, examples, concave/convex functions, point of inflection.

(4) Sketching of graphs of functions using properties.

Reference books:

1. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.

2. James Stewart, Calculus, Third Edition, Brooks/ Cole Publishing company, 1994.

3. T. M. Apostol, Calculus, Vol I, Wiley And Sons (Asia) Pte. Ltd.

4. Sudhir Ghorpade and Balmohan Limaye, A course in Calculus and Real Analysis, Springer International Ltd, 2000.

Additional Reference:

1. Richard Courant and Fritz John, A Introduction to Calculus and Analysis, Volume-I, Springer.
2. Ajit Kumar and S. Kumaresan, A Basic course in Real Analysis, CRC Press, 2014.
3. K. G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
4. G. B. Thomas, Calculus, 12th Edition 2009

USMT 202: DISCRETE MATHEMATICS

Unit I: Preliminary Counting (15 Lectures)

(1) Finite and infinite sets, countable and uncountable sets examples such as $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{N} \times \mathbb{N}$, $\mathbb{Q}$ $(0,1)$, $\mathbb{R}$.

(2) Addition and multiplication Principle, counting sets of pairs, two ways counting.

(3) Stirling numbers of second kind. Simple recursion formulae satisfied by $S(n,k)$ for $k = 1, 2, \cdots, n-1, n$.

(4) Pigeonhole principle simple and strong form and examples, its applications to geometry.

Unit II: Advanced Counting (15 Lectures)

(1) Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems.

(2) Binomial and Multinomial Theorem, Pascal identity, examples of standard identities such as the following with emphasis on combinatorial proofs.

- $\displaystyle\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$
- $\displaystyle\sum_{i=0}^{k} \binom{k}{i}^2 = \binom{2k}{k}$
- $\displaystyle\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$
- $\displaystyle\sum_{i=0}^{n} \binom{n}{i} = 2^n$

(3) Non-negative integer solutions of equation $x_1 + x_2 + \cdots + x_k = n$.

(4) Principal of inclusion and exclusion, its applications, derangements, explicit formula for d_n, deriving formula for Euler's function $\phi(n)$.

Unit III: Permutations and Recurrence relation (15 lectures)

(1) Permutation of objects, S_n, composition of permutations, results such as every permutation is a product of disjoint cycles, every cycle is a product of transpositions, signature of a permutation, even and odd permutations, cardinality of S_n, A_n.

1. Duration: The examinations shall be of 2 and $\frac{1}{2}$ hours duration.
2. Theory Question Paper Pattern:
   a) There shall be FOUR questions. The first three questions Q1, Q2, Q3 shall be of 20 marks, each based on the units I, II, III respectively. The question Q4 shall be of 15 marks based on the entire syllabus.
   b) All the questions shall be compulsory. The questions Q1, Q2, Q3, Q4 shall have internal choices within the questions. Including the choices, the marks for each question shall be 25-27.
   c) The questions Q1, Q2, Q3, Q4 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.
3. **Semester End Examinations Practicals:**
   At the end of the Semesters I & II Practical examinations of three hours duration and 100 marks shall be conducted for the courses USMTP01, USMTP02.
   In semester I, the Practical examinations for USMT101 and USMT102 are held together by the college.
   In Semester II, the Practical examinations for USMT201 and USMT202 are held together by the college.

   Paper pattern: The question paper shall have two parts A and B.
   Each part shall have two Sections.

   Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions (04 objective questions from each unit) ($8 \times 3 = 24$ Marks).

   Section II Problems: Attempt any Two out of Three (01 descriptive question from each unit) ($8 \times 2 = 16$ Marks).

Practical Course	Part A	Part B	Marks out of	duration
USMTP01	Questions from USMT101	Questions from USMT102	80	3 hours
USMTP02	Questions from USMT201	Questions from USMT202	80	3 hours

Marks for Journals and Viva:
For each course USMTP01 (USMT101, USMT102) and USMTP02 (USMT201, USMT202):

1. Journal: 10 marks (5 marks for each journal).
2. Viva: 10 marks.

Each Practical of every course of Semester I and II shall contain at least 10 objective questions and at least 6 descriptive questions.
A student must have a certified journal before appearing for the practical examination.
In case a student does not posses a certified journal he/she will be evaluated for 80 marks.
He/she is not qualified for Journal + Viva marks.

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